

# Declarative Belief Set Merging using Merging Plans

Christoph Redl   Thomas Eiter   Thomas Krennwallner

{redl,eiter,tkren}@kr.tuwien.ac.at



TECHNISCHE  
UNIVERSITÄT  
WIEN  
Vienna University of Technology



January 24, 2011

# Outline

- 1 Motivation
- 2 Merging Framework
- 3 Prototype Implementation MELD
- 4 Application and Discussion
- 5 Conclusion

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## Usage of Multiple Knowledge Bases

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## Examples

- Judgment aggregation (discussed later)
- Merging of decision diagrams

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- **Belief:** atomic formula or a negated atomic formula

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## Definition (Knowledge Bases)

- We **abstract from a concrete language** for knowledge bases  $KB$
- **Knowledge bases** are **identified with assigned collections of belief sets** (their “semantics”):  $BS(KB) \subseteq \mathcal{A}(\Sigma)$

# Belief Sets and Knowledge Bases

## Example

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- Classically entailed literals:

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# Merging Task

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  - logical inconsistencies

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- Different sources may use different vocabularies
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$$\mathcal{B}'_i = \mu_i(\mathcal{B}_i)$$

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Formally: A **belief set conversion** is a function

$$\mu_i : 2^{\mathcal{A}(\Sigma^{KB_i})} \rightarrow 2^{\mathcal{A}(\Sigma^C)}, 1 \leq i \leq n$$

s.t.  $\mathcal{B}'_i = \mathcal{B}'_j$  iff they are considered to represent the same information

# Mismatch 1: Language Incompatibilities

## Example (continued)

$$\mu_1(\mathcal{B}) = \mathcal{B},$$

$$\mu_2(\mathcal{B}) = \{ \{ \text{degree}(X, \text{“MSc”}) \mid \text{deg}(X, \text{“Master of Science”}) \in B \} \cup \\ \{ \text{degree}(X, Y) \mid \text{deg}(X, Y) \in B, Y \neq \text{“Master of Science”} \} \mid B \in \mathcal{B} \};$$

## Mismatch 2: Logical Inconsistencies

### Definition (Integrity Constraints)

Application-dependent integrity constraints are abstractly modeled as

$$\mathcal{C} \subseteq 2^{\mathcal{A}(\Sigma^C)},$$

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- We introduce **merging operators**

$$\circ^{n,m} : \underbrace{\left(2^{\mathcal{A}(\Sigma^C)}\right)^n}_{\text{collections of belief sets}} \rightarrow 2^{\mathcal{A}(\Sigma^C)}$$

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- We introduce **merging operators**
- Maps  $n$  collections of belief sets to a new, integrated collection

$$\circ^{n,m} : \underbrace{\left(2^{\mathcal{A}(\Sigma^C)}\right)^n}_{\text{collections of belief sets}} \times \underbrace{\mathcal{D}_1 \times \dots \times \mathcal{D}_m}_{\text{additional parameters}} \rightarrow 2^{\mathcal{A}(\Sigma^C)}$$

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**Operator definition:** (binary, no parameter, i.e.,  $n = 2, m = 0$ )

$$\circ_{\cup}^{2,0}(\mathcal{B}_1, \mathcal{B}_2) = \{B_1 \cup B_2 \mid B_1 \in \mathcal{B}_1, B_2 \in \mathcal{B}_2, \nexists A : \{A, \neg A\} \subseteq (B_1 \cup B_2)\},$$

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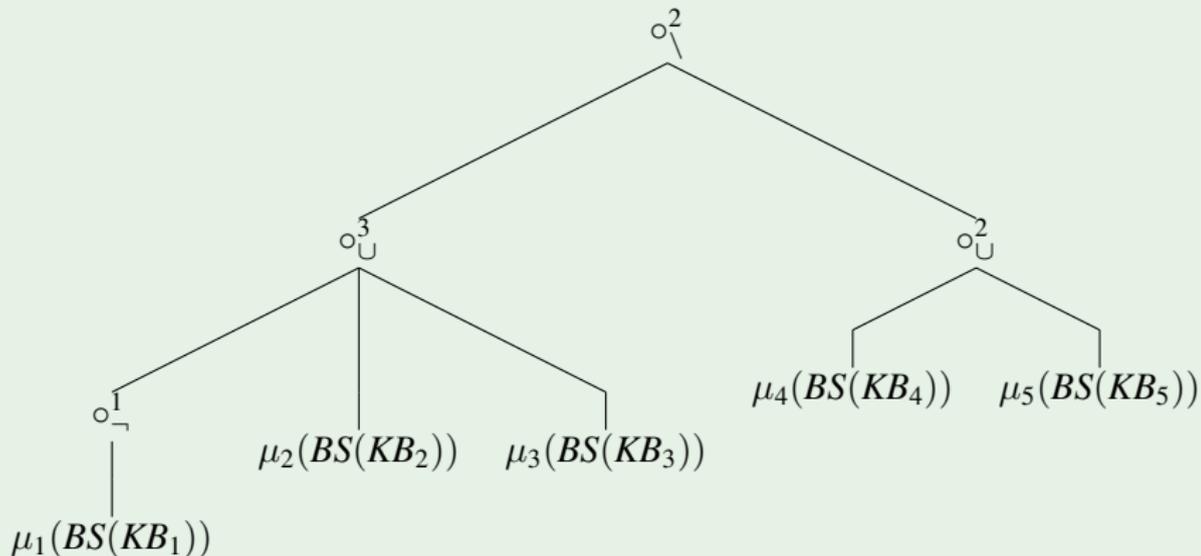
**Application:**

- $\mathcal{B}_1 = \{\{a, b, c\}, \{\neg a, c\}\}$
- $\mathcal{B}_2 = \{\{\neg a, d\}, \{c, d\}\}$
- $\circ_{\cup}^{2,0}(\mathcal{B}_1, \mathcal{B}_2) = \{\{a, b, \neg a, d\}, \{a, b, c, d\}, \{\neg a, c, d\}, \{\neg a, c, d\}\}$

# Merging Plans

Hierarchical arrangement of operators:

## Example



# Merging Plans

## Definition (Merging Plans)

The set  $\mathcal{M}_{KB,\Omega}$  of merging plans over knowledge bases  $KB = KB_1, \dots, KB_n$  and a set  $\Omega = \{\circ_1, \dots, \circ_n\}$  of operators is the smallest set such that

- (i) each  $M \in KB$ , called *atomic* merging plan, is in  $\mathcal{M}_{KB,\Omega}$ ;
- (ii) if  $\circ_i^{n,m} \in \Omega$ ,  $s_j \in \mathcal{M}_{KB,\Omega}$  and  $a_k \in \mathcal{D}_i$  for  $1 \leq j \leq n$ ,  $1 \leq k \leq m$ , then  $(\circ_i^{n,m}, s_1, \dots, s_n, a_1, \dots, a_m) \in \mathcal{M}_{KB,\Omega}$ .

## Example (continued)

$$M = (\circ_{\surd}^2, (\circ_{\cup}^3, (\circ_{\neg}^1, KB_1), KB_2, KB_3), (\circ_{\cup}^2, KB_4, KB_5)).$$

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## Definition (Merging Task Result)

The *result of a merging task*  $T = \langle KB, \Sigma^C, \mu, \Omega, M \rangle$ , denoted as  $\llbracket T \rrbracket$ , is

$$\llbracket T \rrbracket = \begin{cases} [\mu_i(BS(M))]_{\Sigma_p^C}, & \text{if } M \in KB, \\ [\circ^{n,m}(\llbracket T_1 \rrbracket, \dots, \llbracket T_n \rrbracket, a_1, \dots, a_m)]_{\Sigma_p^C}, & \text{if } M = (\circ^{n,m}, s_1, \dots, s_n, a_1, \dots, a_m), \end{cases}$$

where  $[\mathcal{B}]_{\Sigma_p^C} = \{ \{ p(a_1, \dots, a_n) \in BS \mid p = (\neg)p', p' \in \Sigma_p^C \} \mid BS \in \mathcal{B} \}$  denotes the projection of  $\mathcal{B}$  to the atoms over  $\Sigma_p^C$ , and  $T_i = \langle KB, \Sigma^C, \mu, \Omega, s_i \rangle$ ,  $1 \leq i \leq n$ .

## Intuition

- The result of a merging plan will be defined as the collection of belief sets delivered by the topmost operator

# Merging Plans

## Example (continued)

$$M = (\circ_{\setminus}^2, (\circ_{\cup}^3, (\circ_{\neg}^1, KB_1), KB_2, KB_3), (\circ_{\cup}^2, KB_4, KB_5)).$$

Let

$$KB_1 = \{a., b.\}, KB_2 = \{x., y.\}, KB_3 = \{\neg a., c.\}, KB_4 = \{a., x.\}, KB_5 = \{c., x., y.\}$$

under answer-set semantics ( $x.$  is an abbreviation for  $x \leftarrow .$ )

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Evaluation:

$$\begin{aligned} \llbracket \langle \{KB_1, \dots, KB_5\}, \Sigma^C, \mu_{id}, \Omega, M \rangle \rrbracket &= \\ \circ_{\vee}^2 ( \llbracket (\circ_{\cup}^3, (\circ_{\neg}^1, KB_1), KB_2, KB_3) \rrbracket, \llbracket (\circ_{\cup}^2, KB_4, KB_5) \rrbracket ) &= \\ \circ_{\vee}^2 ( \circ_{\cup}^3 ( \llbracket (\circ_{\neg}^1, KB_1) \rrbracket, \llbracket KB_2 \rrbracket, \llbracket KB_3 \rrbracket ) , \llbracket (\circ_{\cup}^2, KB_4, KB_5) \rrbracket ) &= \\ \dots = \circ_{\vee}^2 ( \{ \{ \neg a, \neg b, c, x, y \} \}, \{ \{ a, c, x, y \} \} ) &= \{ \{ \neg a, \neg b \} \}. \end{aligned}$$

$\llbracket M \rrbracket$  is an abbreviation for  $\llbracket \{P_1, \dots, P_5\}, \Sigma^C, \mu_{id}, \Omega, M \rrbracket$

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rules under HEX-semantics; query the source (1) in the body; derive atoms over common signature (2) in the head

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# Merging Task Language

## Example: `merging.mt`

[common signature]

```
predicate: a/0;
predicate: b/0;
predicate: c/0;
predicate: p/1;
predicate: q/3;
```

[belief base]

```
name:bb1;
mapping: "some_rule.";      % query external source here
mapping: "q(X, Y, Z) :- &rdf[...](X, Y, Z).";
```

[belief base]

```
name:bb2;
source: "some_program.hex";  % or within this program
```

...

# Merging Task Language

## Example: `merging.mt` (ctn'd)

```
[merging plan]
{  operator: setminus;
  {
    operator: union;
    {
      operator: neg;
      {bb1};
    };
    {bb2};
    {bb3};
  };
  {
    operator: union;
    {bb4};
    {bb5};
  };
}
```

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# Outline

- 1 Motivation
- 2 Merging Framework
- 3 Prototype Implementation MELD
- 4 Application and Discussion**
- 5 Conclusion

# Judgment Aggregation

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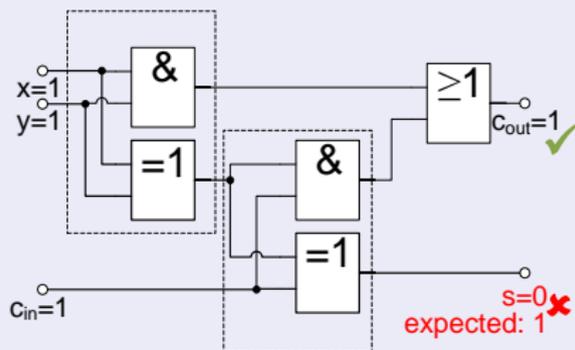
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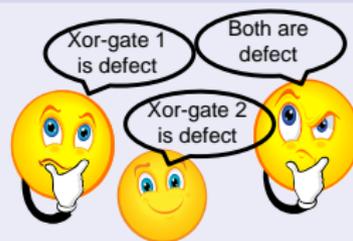
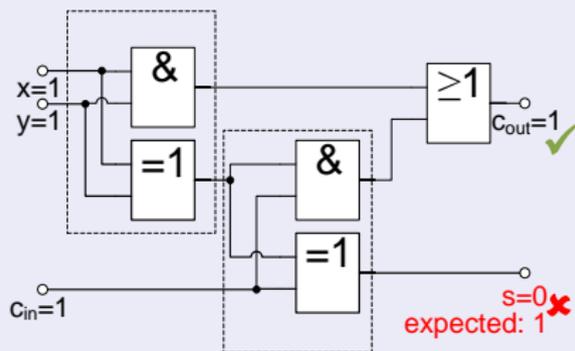


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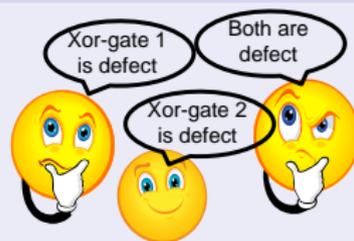
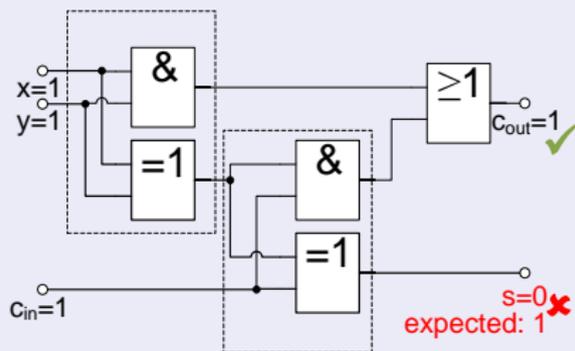
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- it is still be an explanations
- it is *similar* to individual opinions

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## Advantages

- Reusing of operators
- Evaluating different operators empirically
- Automatic recomputation of result
- Release user from routine tasks

# References



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