Eliminating Unfounded Set Checking for HEX-Programs

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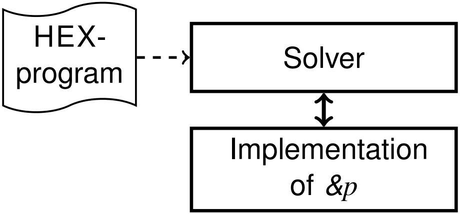
Motivation

HEX-programs

- extend ordinary ASP programs by external atoms *&p*
- allows to access external knowledge



Answer Set Computation

Issue: efficient computation of Answer Sets for HEX-programs

- Faber-Leone-Pfeifer (FLP) semantics [Faber *et al.*, 2011] (minimal models of FLP-reduct)
- issue of nonmonotonic external atoms with recursion
- reasoning from Horn-programs with poly external atoms is Σ_2^p-hard
- thus: answer set checking needs special care

Answer Set Computation

Issue: efficient computation of Answer Sets for HEX-programs

- simple search for smaller models does not scale
- Unfounded Sets (UFS): used to reduce FLP answer set checking to a search for UFS (implemented as a SAT problem) [Eiter *et al.*, 2012b]
- Here: find syntactic criterions to avoid UFS checking

HEX-Programs

Definition (HEX-programs)

A HEX-program consists of rules of form

$$a_1 \vee \cdots \vee a_n \leftarrow b_1, \ldots, b_m, \text{not } b_{m+1}, \ldots, \text{not } b_n,$$

with classical literals a_i, and classical literals or an external atoms b_j.

Definition (External Atoms)

An external atom is of the form

$$\&p[q_1, \ldots, q_k](t_1, \ldots, t_l),$$

p ... external predicate name

q_i ... predicate names or constants

t_j ... terms

Semantics: $1 + k + l$-ary Boolean oracle function $f_{\&p}$:
$\&p[q_1, \ldots, q_k](t_1, \ldots, t_l)$ is true under assignment $\mathbf{A}$ iff
$f_{\&p}(\mathbf{A}, q_1, \ldots, q_k, t_1, \ldots, t_l) = 1$.

Examples

The $\&rdf$ External Atom

- Input: URL
- Output: Set of triplets from RDF file

External knowledge base is a set of RDF files on the web:

$$addr(\texttt{http://.../data1.rdf}).$$
$$addr(\texttt{http://.../data2.rdf}).$$
$$bel(X,Y) \leftarrow addr(U), \&rdf[U](X,Y,Z).$$

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$\&diff[p,q](X)$: all elements X, which are in p but not in q:

$$\begin{array}{rcl} dom(X) & \leftarrow & \#int(X). \\ nsel(X) & \leftarrow & dom(X), \&diff[dom, sel](X). \\ sel(X) & \leftarrow & dom(X), \&diff[dom, nsel](X). \\ & \leftarrow & sel(X1), sel(X2), sel(X3), X1 \neq X2, X1 \neq X3, X2 \neq X3. \end{array}$$

Evaluation Method

Translation Approach

HEX-Program Π:

$$p(c_1).\ dom(c_1).\ dom(c_2).\ dom(c_3).$$
$$p(X) \leftarrow dom(X), \&empty[p](X).$$

Guessing program $\hat{\Pi}$:

$$p(c_1).\ dom(c_1).\ dom(c_2).\ dom(c_3).$$
$$p(X) \leftarrow dom(X), e_{\&empty[p]}(X).$$
$$e_{\&empty[p]}(X) \vee \neg e_{\&empty[p]}(X) \leftarrow dom(X).$$

8 candidates, e.g.:

$\{\mathbf{T}p(c_1), \mathbf{T}p(c_2), \mathbf{T}dom(c_1), \mathbf{T}dom(c_2), \mathbf{T}dom(c_3),$
$\mathbf{F}e_{\&empty[p]}(c_1), \mathbf{T}e_{\&empty[p]}(c_2), \mathbf{F}e_{\&empty[p]}(c_3)\}$

Compatibility check: passed $\Rightarrow$ compatible set

Minimality Criterion

Definition (FLP-Reduct [Faber *et al.*, 2011])

For an interpretation $\mathbf{A}$ over a program Π, the FLP-reduct $f\Pi^{\mathbf{A}}$ of Π wrt. $\mathbf{A}$ is the set $\{r \in \Pi \mid \mathbf{A} \models b, \text{ for all } b \in B(r)\}$ of all rules whose body is satisfied under $\mathbf{A}$.

Definition (Answer Set)

An interpretation $\mathbf{A}$ is an answer set of program Π iff it is a subset-minimal model of the FLP reduct $f\Pi^{\mathbf{A}}$.

Example

$$dom(a). \quad dom(b). \quad p(a) \leftarrow dom(a), \&g[p](a). \quad p(b) \leftarrow dom(b), \&g[p](b).$$

where $\&g$ implements the following mapping:

$\emptyset \mapsto \{b\}; \{a\} \mapsto \{a\}; \{b\} \mapsto \emptyset; \{a,b\} \mapsto \{a,b\}$

$\mathbf{A} = \{\mathbf{T}dom(a), \mathbf{T}dom(b), \mathbf{T}p(a), \mathbf{F}p(b)\}$ is a model, but not subset-minimal model of $f\Pi^{\mathbf{A}}$: $dom(a).\ dom(b).\ p(a) \leftarrow dom(a), \&g[p](a)$

Using Unfounded Sets

Definition (Unfounded Set [Faber, 2005])

A set of atoms X is an unfounded set of Π wrt. (partial) assignment $\mathbf{A}$, iff for all $a \in X$ and all $r \in \Pi$ with $a \in H(r)$, at least one of (1)–(3) holds:

1. $\mathbf{A} \not\models B(r)$
2. $\mathbf{A} \dot{\cup} \neg.X \not\models B(r)$
3. $\mathbf{A} \models h$ for some $h \in H(r) \setminus X$

(where $\mathbf{A} \dot{\cup} \neg.X = \{\mathbf{T}a \in \mathbf{A} \mid a \notin X\} \cup \{\mathbf{F}a \in \mathbf{A}\} \cup \{\mathbf{F}a \mid a \in X\}$)

Definition (Unfounded-free Assignments)

An assignment $\mathbf{A}$ is unfounded-free wrt. program Π, iff there is no unfounded set X of Π wrt. $\mathbf{A}$ such that $\mathbf{T}a \in \mathbf{A}$ for some $a \in X$.

Theorem (FLP Answer sets)

A model $\mathbf{A}$ of a program Π is is an answer set iff it is unfounded-free.

Atom Dependency Graph

Definition (Atom Dependency)

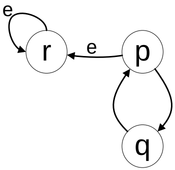
For a ground program Π and ground atoms $p(\mathbf{c})$, $q(\mathbf{d})$, we say:

(i) $p(\mathbf{c})$ depends on $q(\mathbf{d})$ ($p(\mathbf{c}) \rightarrow q(\mathbf{d})$) iff for some rule $r \in \Pi$ we have $p(\mathbf{c}) \in H(r)$ and $q(\mathbf{d}) \in B(r)$

(ii) $p(\mathbf{c})$ depends externally on $q(\mathbf{d})$ ($p(\mathbf{c}) \rightarrow_e q(\mathbf{d})$) iff for some rule $r \in \Pi$ we have $p(\mathbf{c}) \in H(r)$ and there is a $\&g[q_1, \ldots, q_n](\mathbf{d}) \in B^+(r) \cup B^-(r)$ with $q_i = q$ for some $i \in \{1, \ldots, n\}$.

Example

$\Pi = \{r \leftarrow \&id[r](); \quad p \leftarrow \&id[r](); \quad p \leftarrow q; \quad q \leftarrow p\}$



Cuts

Definition (Cut)

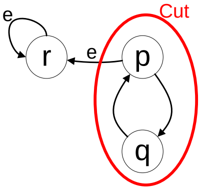
Let U be an UFS of Π wrt. $\mathbf{A}$. A set of atoms $C \subseteq U$ is a cut, if

(i) For all $a \in C, b \in U$: $b \not\to_e a$, and

(ii) For all $a \in C, b \in U \setminus C$: $b \not\to a$ and $a \not\to b$.

Lemma (Unfounded Set Reduction Lemma)

Let U be an UFS of Π wrt. $\mathbf{A}$ and let C be a cut. Then $Y = U \setminus C$ is an unfounded set of Π wrt. $\mathbf{A}$.



Example

$\Pi = \{r \leftarrow \&id[r](); \quad p \leftarrow \&id[r](); \quad p \leftarrow q; \quad q \leftarrow p\}$

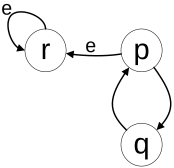
UFS $U = \{p, q, r\}$ wrt. $\mathbf{A} = \{\mathbf{T}p, \mathbf{T}q, \mathbf{T}r\}$

$\Rightarrow$ UFS $U' = \{p, q, r\} \setminus \{p, q\} = \{r\}$ wrt. $\mathbf{A}$

EA-Input Unfoundedness

Lemma (EA-Input Unfoundedness)

Let U be an unfounded set of Π wrt. $\mathbf{A}$. If there are no $x, y \in U$ s.t. $x \rightarrow_e y$, then U is an unfounded set of $\hat{\Pi}$ wrt. $\hat{\mathbf{A}}$.



Example

$\Pi = \{r \leftarrow \&id[r]();\quad p \leftarrow \&id[r]();\quad p \leftarrow q;\quad q \leftarrow p\}$

UFS $U_1 = \{p, q\}$ wrt. $\mathbf{A}' = \{\mathbf{T}p, \mathbf{T}q, \mathbf{F}r\}$ is already detected when $\hat{\Pi} = \{e_{\&id[r]}() \vee \neg e_{\&id[r]}() \leftarrow;\quad r \leftarrow e_{\&id[r]}();\quad p \leftarrow e_{\&id[r]}();\quad p \leftarrow q;\quad q \leftarrow p\}$ is evaluated

UFS $U_2 = \{p, q, r\}$ wrt. $\mathbf{A}'' = \{\mathbf{T}p, \mathbf{T}q, \mathbf{T}r\}$ is not detected during model generation phase of the ordinary part as $p, r \in U_2$ and $p \rightarrow_e r$

E-Cycles

Definition (Cycle and E-Cycle)

A cycle under a binary relation $\circ$ is a sequence of elements $C = c_0, \ldots, c_{n+1}$ $(n \geq 0)$ s.t. $(c_i, c_{i+1}) \in \circ$ for all $i \in \{0, \ldots, n\}$ and $c_0 = c_{n+1}$.

Let $\rightarrow^d = \rightarrow \cup \leftarrow \cup \rightarrow_e$ ($\leftarrow$ is the inverse of $\rightarrow$).
A cycle $c_0, \ldots, c_{n+1}$ in $\rightarrow^d$ is called an e-cycle, iff it contains e-edges.

Proposition (Relevance of e-cycles)

Suppose U is an unfounded set of Π wrt. $\mathbf{A}$ which contains no e-cycle under $\rightarrow^d$. Then there exists an unfounded set of $\hat{\Pi}$ wrt. $\hat{\mathbf{A}}$.

Corollary

If there is no e-cycle under $\rightarrow^d$ and $\hat{\Pi}$ has no unfounded set wrt. $\hat{\mathbf{A}}$, then $\mathbf{A}$ is unfounded-free for Π.

E-Cycles

Example (Programs without E-Cycles)

$\Pi_1 = \{out(X) \leftarrow \&diff[set_1, set_2](X)\} \cup F$ ($F \ldots$ set of facts)

$\Pi_2 = \{str(Z) \leftarrow dom(Z), str(X), str(Y), \text{not } \&concat[X, Y](Z)\}$

E-Cycles

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$\Pi_2 = \{str(Z) \leftarrow dom(Z), str(X), str(Y), \text{not } \&concat[X, Y](Z)\}$

Proposition (Unfoundedness of Cyclic Input Atoms)

If U is an unfounded set of Π wrt. $\mathbf{A}$ and U contains no cyclic input atoms, then $\hat{\Pi}$ has an unfounded set wrt. $\hat{\mathbf{A}}$.

Program Decomposition

Let $\mathcal{C}$ be a partitioning of the ordinary atoms $A(\Pi)$ of Π into $\subseteq$-maximal strongly connected components under $\rightarrow \cup \rightarrow_e$.

Definition (Associated Programs)

For each $C \in \mathcal{C}$, the program associated with C is defined as

$$\Pi_C = \{r \in \Pi \mid H(r) \cap C \neq \emptyset\} \ .$$

Proposition

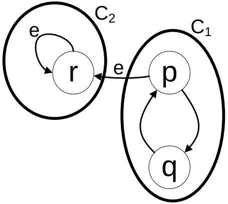
Let U be a nonempty unfounded set of Π wrt. $\mathbf{A}$. Then for some Π_C with $C \in \mathcal{C}$ we have that $U \cap C$ is an unfounded set of Π_C wrt. $\mathbf{A}$.

Proposition

Let U be a nonempty unfounded set of Π_C wrt. $\mathbf{A}$ such that $U \subseteq C$. Then U is an unfounded set of Π wrt. $\mathbf{A}$.

Program Decomposition Example

$$\Pi = \{r \leftarrow \&id[r](); \quad p \leftarrow \&id[r](); \quad p \leftarrow q; \quad q \leftarrow p\}$$



$\mathcal{C} = \{C_1, C_2\}$ with $C_1 = \{p, q\}$ and $C_2 = \{r\}$
$\Pi_{C_1} = \{p \leftarrow \&id[r](); p \leftarrow q; q \leftarrow p\}$
$\Pi_{C_2} = \{r \leftarrow \&id[r]()\}$.

Let $U = \{p, q, r\}$ be an UFS wrt. $\mathbf{A} = \{\mathbf{T}p, \mathbf{T}q, \mathbf{T}r\}$
Then $U \cap \{r\} = \{r\}$ is also an unfounded set of Π_{C_2} wrt. $\mathbf{A}$

Experiments

Implementation: dlvhex [Eiter *et al.*, 2012a]

Argumentation

#args	first answer set					all answer sets				
	standard approach		new approach			standard approach		new approach		
	timeouts	avg	timeouts	avg	gain	timeouts	avg	timeouts	avg	gain
5	0	1,09	0	1,07	2,21%	0	1,70	0	1,56	9,21%
6	0	2,40	0	2,30	4,58%	0	4,58	0	3,74	22,58%
7	0	5,58	0	5,33	4,68%	0	15,66	0	11,28	38,78%
8	0	14,26	0	12,74	11,99%	3	71,06	2	39,32	80,71%
9	0	39,82	0	33,57	18,63%	16	174,99	8	106,34	64,55%
10	2	126,54	0	80,00	58,18%	40	278,98	16	214,81	29,87%

Conclusion

Conclusion

- Decision criterion for avoiding the UFS check
- Based on concept of e-cycles
- Modular application via program decomposition
- Benchmark results are promising

Future Work

- Other syntactic criteria
- Use semantic information

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